Numerical simulation of epithelial tissues using FENE-P viscoelastic fluid model

M. Renard[1] P. Saramito[1] H. Delanoë-Ayari[2] I. Cheddadi[3] F. Graner[4]

[1] LJK - Univ. Grenoble Alpes
[2] iLM - Univ. Lyon 1
[3] TIMC - Univ. Grenoble Alpes
[4] MSC - Univ. Paris Cité

AERC Lyon-France
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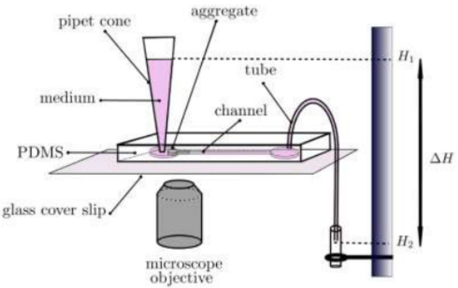
Introduction

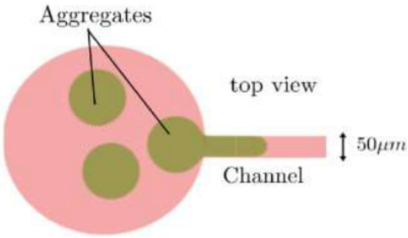
Epithelial tissues

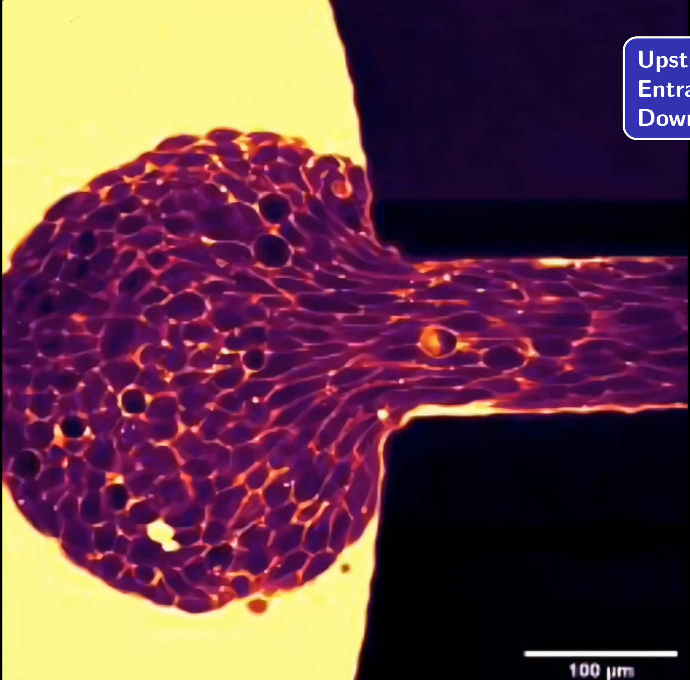
- Role: wound healing, morphogenesis, ...
- Active processes: division, growth, migration ...
- Passive processes: stress, deformation → **rheology**

Experiments *[Tlili et al. 2022]*

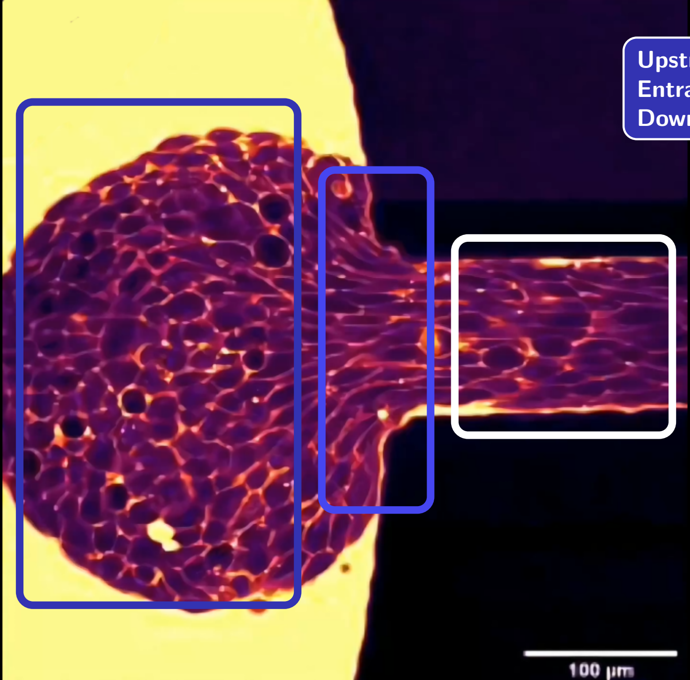
- Abrupt contraction
- Heterogeneous flow
- Large deformations







Upstream: rest, circular
Entrance: stretch, shear
Downstream: relaxation



Upstream: rest, circular
Entrance: stretch, shear
Downstream: relaxation

Modeling needs

Unknowns

Pressure	p	Non-Newtonian
Velocity	$\boldsymbol{u}$	
Conformation	$\boldsymbol{c}$	

Parameters

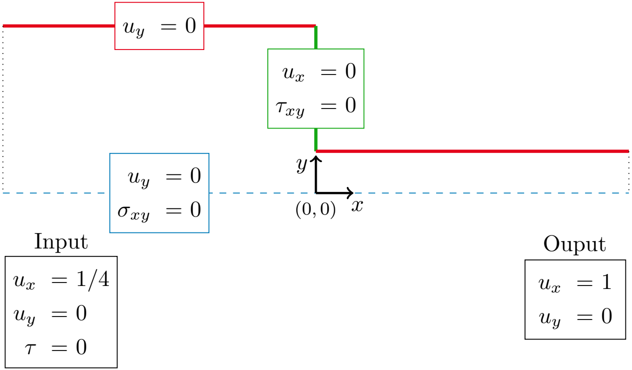
Viscosity	α
Weissenberg	We
Reynolds	Re

Assumptions

1. Isotropic rest state
2. Re $\ll$ 1 (small velocity)
3. Depth invariance $\rightarrow$ 2D

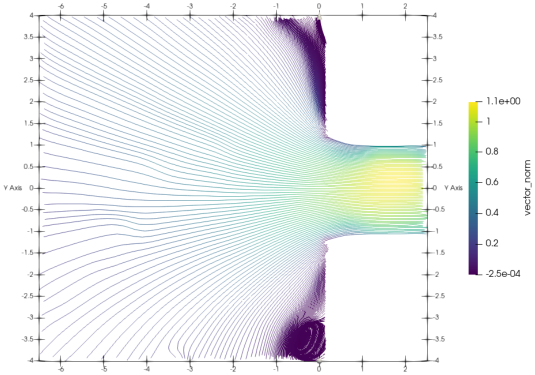
Features

1. Stationary solutions
2. Slip boundaries
3. Abrupt geometry

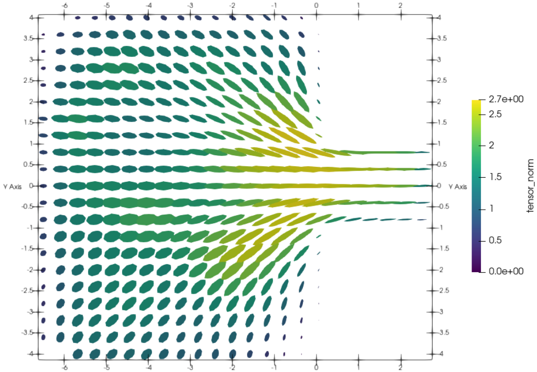


Experimental data fields

Velocity & conformation



Velocity streamlines



Conformation field

Identify: Cell shape $\longleftrightarrow$ Ellipse

Navier-Stokes is not enough !

Oldroyd-B model

[Oldroyd 1950]

$\boldsymbol{c}$	Conformation
$\boldsymbol{u}$	Velocity
p	Pressure
–	————
We	Weissenberg
α	Polymeric viscosity

Find $\boldsymbol{c}, \boldsymbol{u}$ and p defined in Ω such that

$$\begin{cases} \mathrm{We}\,\overset{\triangledown}{\boldsymbol{c}} + \boldsymbol{c} & = \dfrac{\alpha}{\mathrm{We}}\mathbb{I}_d \\ \cancel{\mathrm{Re}\dfrac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}} - \mathrm{div}\,(\boldsymbol{\sigma}) & = 0 \\ \mathrm{div}(\boldsymbol{u}) & = 0 \\ \boldsymbol{\sigma} & = \boldsymbol{c} - \dfrac{\alpha}{\mathrm{We}}\mathbb{I}_d + 2(1-\alpha)D(\boldsymbol{u}) - p \cdot \mathbb{I}_d \end{cases}$$

Assets

- Linear, large deformations
- Few parameters

Shortcomings

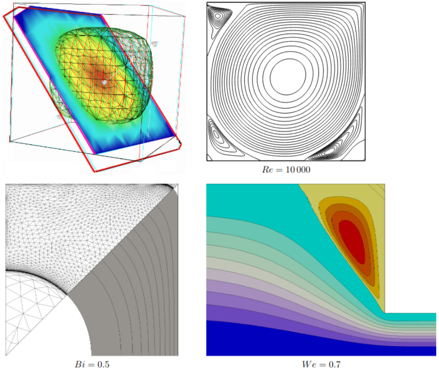
- Unbounded $\mathrm{Tr}(\boldsymbol{c})$ *[Avesiani - internship 2023]*
- No global existence result *[Renardy, Thomases 2022]*

Numerical Method

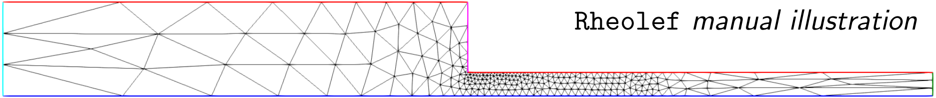
Main items

- Finite Elements Method
- Damped Newton algorithm
- Log of conformation formulation
- Parallel Computing & `C++`

`Rheolef 7.2` *[Saramito 2003-2025]*



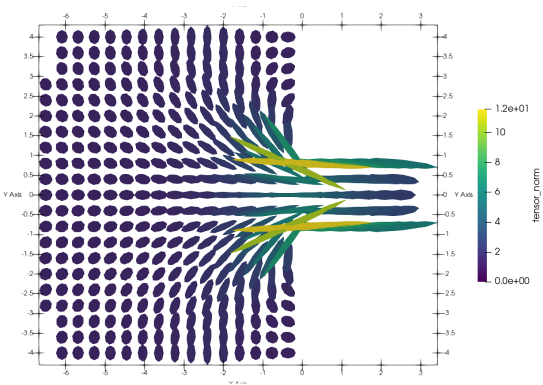
`Rheolef` *manual illustration*



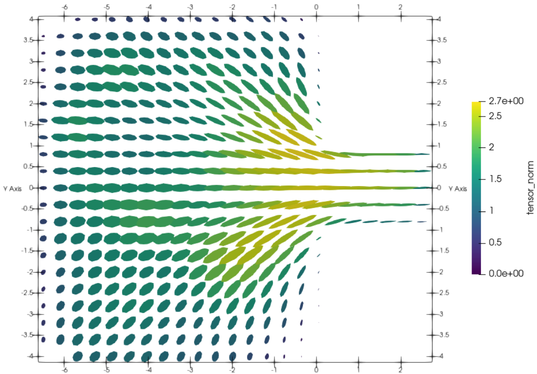
Example mesh of contraction geometry

Oldroyd-B vs. Experiments

Conformation maps



Model



Experiment

Entrance stretching too important

Need elasticity limitation !

FENE-P model

[Bird et al. 1980]

Find $\boldsymbol{c}, \boldsymbol{u}$ and p defined in Ω such that

$$\begin{cases} \mathrm{We}\, \overset{\triangledown}{\boldsymbol{c}} + \dfrac{\boldsymbol{c}}{1 - \beta \dfrac{\mathrm{We}}{\alpha} \mathrm{Tr}(\boldsymbol{c})} & = \dfrac{\dfrac{\alpha}{\mathrm{We}} \mathbb{I}_d}{1 - \beta d} \\ \cancel{\mathrm{Re} \dfrac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}} - \mathrm{div}\,(\boldsymbol{\sigma}) & = 0 \\ \mathrm{div}(\boldsymbol{u}) & = 0 \\ \sigma & = \dfrac{\boldsymbol{c}}{1 - \beta \dfrac{\mathrm{We}}{\alpha} \mathrm{Tr}(\boldsymbol{c})} - \dfrac{\dfrac{\alpha}{\mathrm{We}} \mathbb{I}_d}{1 - \beta d} + 2(1-\alpha) D(\boldsymbol{u}) - p \cdot \mathbb{I}_d \end{cases}$$

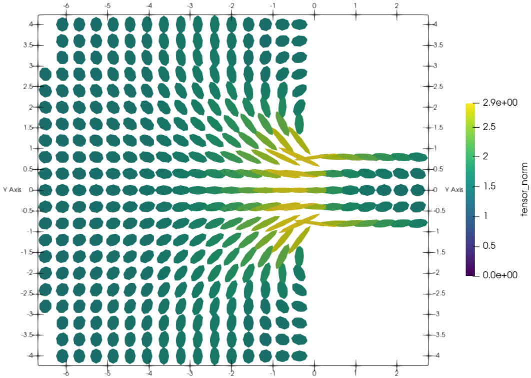
$\boldsymbol{c}$	Conformation
$\boldsymbol{u}$	Velocity
p	Pressure
–	————
We	Weissenberg
α	Polymeric viscosity
β	Extension limiter

Assets

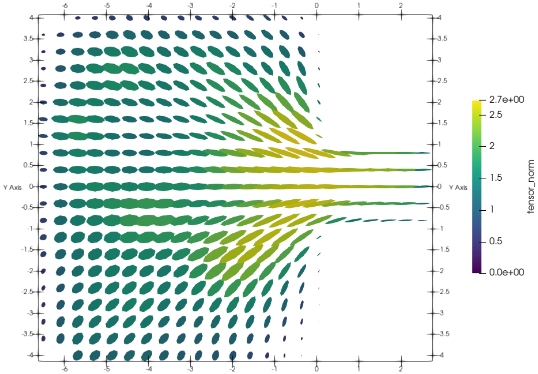
- Large deformations & $\mathrm{Tr}(\boldsymbol{c}) < \frac{\alpha}{\beta \mathrm{We}}$
- Contains Oldroyd-B ($\beta = 0$)
- Global (weak) existence *[Masmoudi 2011]*
- Isotropic rest state $\boldsymbol{c} = \frac{\alpha}{\mathrm{We}} \mathbb{I}_d$

FENE-P vs. Experiments

Conformation maps



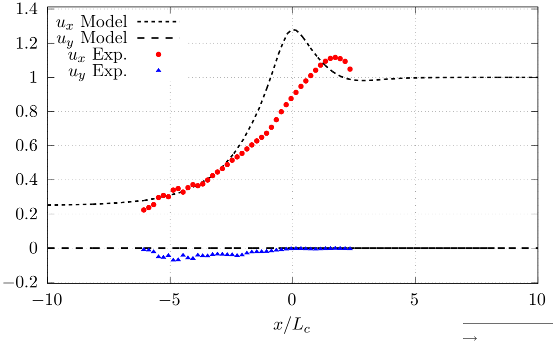
Model



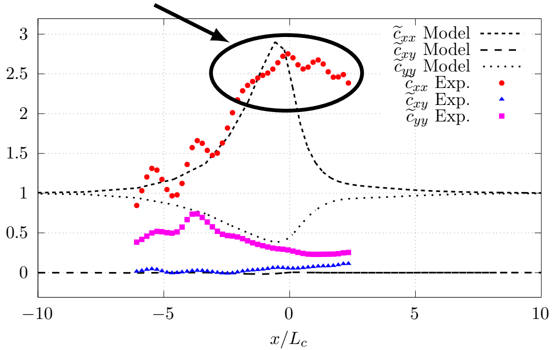
Experiment

Upstream & Entrance ✓

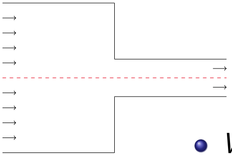
Downstream relaxation not matching !



Velocity



Conformation



- Good agreement upstream
- Downstream plug flow

- *We* number cannot be increased
- **Need for a yield stress**

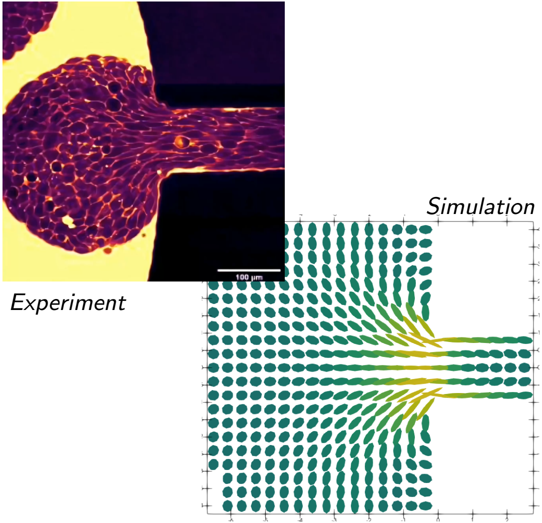
Advances

- Epithelial tissues: viscoelastic behavior
- Oldroyd-B: unbounded extension
- FENE-P: upstream and entrance ✓
- **Agreement for large deformation viscoelasticity**

+ Slip b.c. & HPC implementation

Current Limitation

- Upstream / Downstream states
- **Need for a yield stress**



Experiment

Simulation

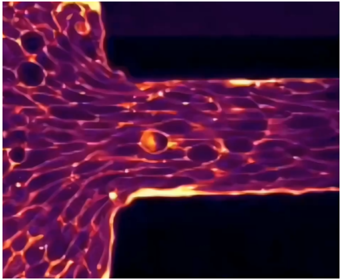
Perspectives

Modeling

- Elastoviscoplastic FENE-P
- Yield stress

Data

- Test other mutants
- Change pressure gradient
- Diversify geometries
- Study cellular density field

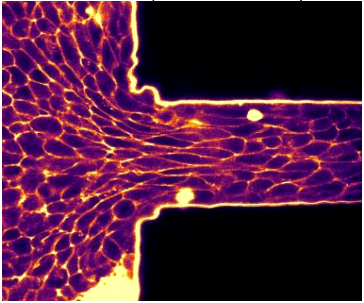


Initial lineage

Change downstream relaxation

[Tlili, Graner, Delanoë-Ayari, 2022]

Mutants (α-catenin-null)



Thank you for listening !