Viscoelastic fluid modeling for biological tissues under large deformations

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Outline

Introduction

Embryonic tissues

- Role: wound healing, morphogenesis, ...
- Active processes: division, growth, migration ...
- Passive processes: stress, deformation $\rightarrow$ **rheology**

Embryonic tissues

- Role: wound healing, morphogenesis, ...
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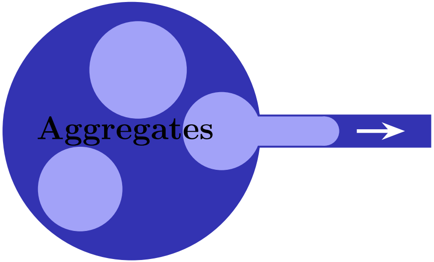
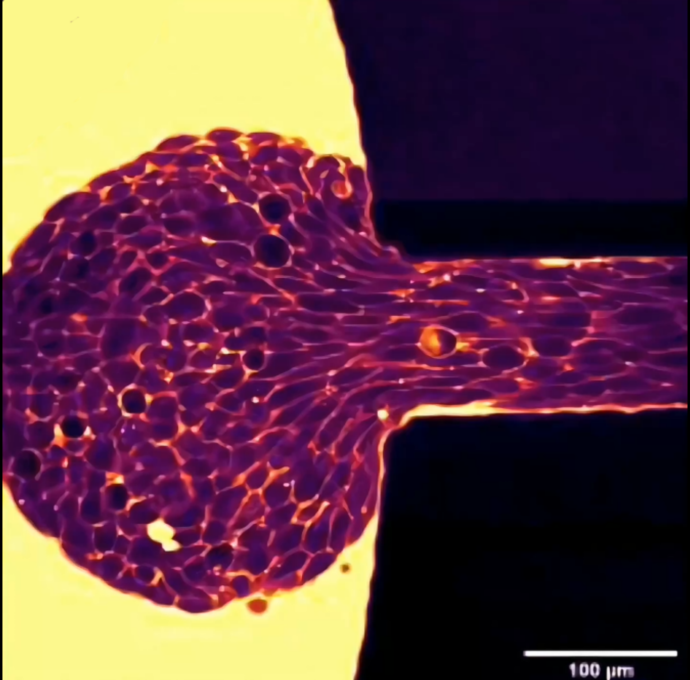
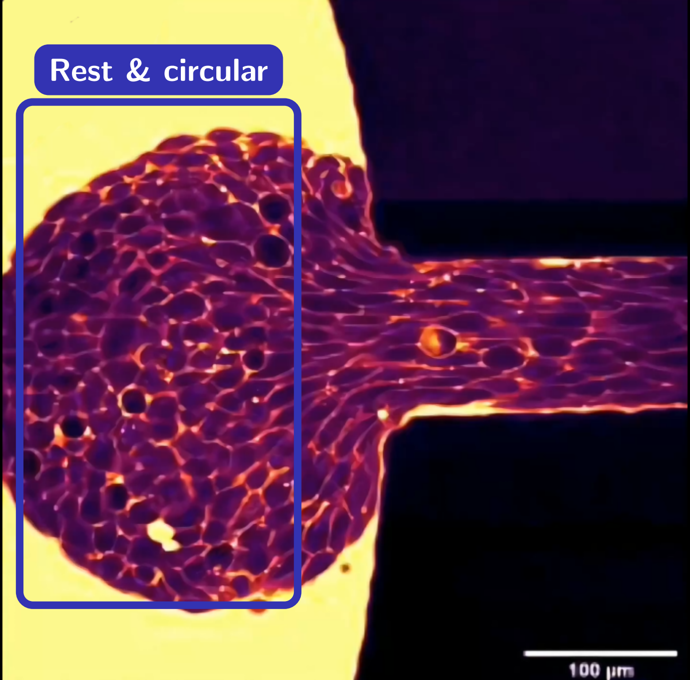
- Passive processes: stress, deformation → **rheology**

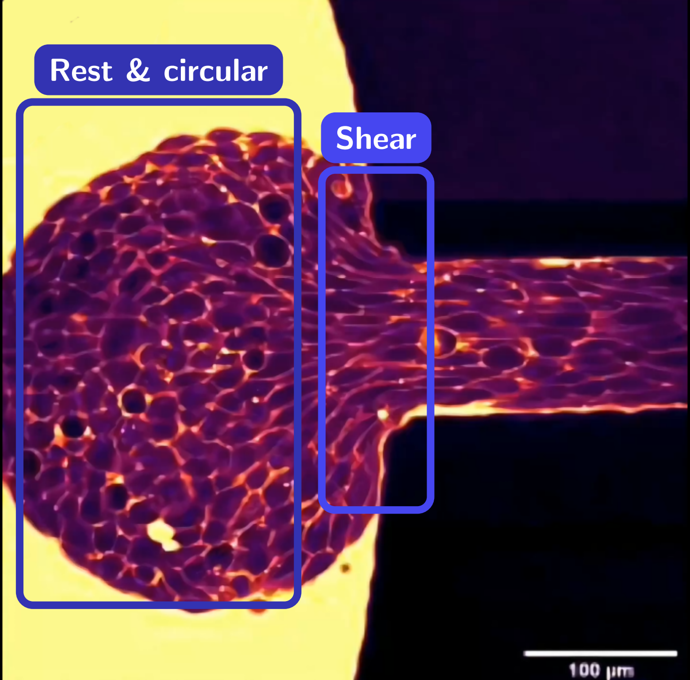
Experiments *[Tlili et al. 2022]*

- Abrupt contraction
- Heterogeneous flow
- Large deformations

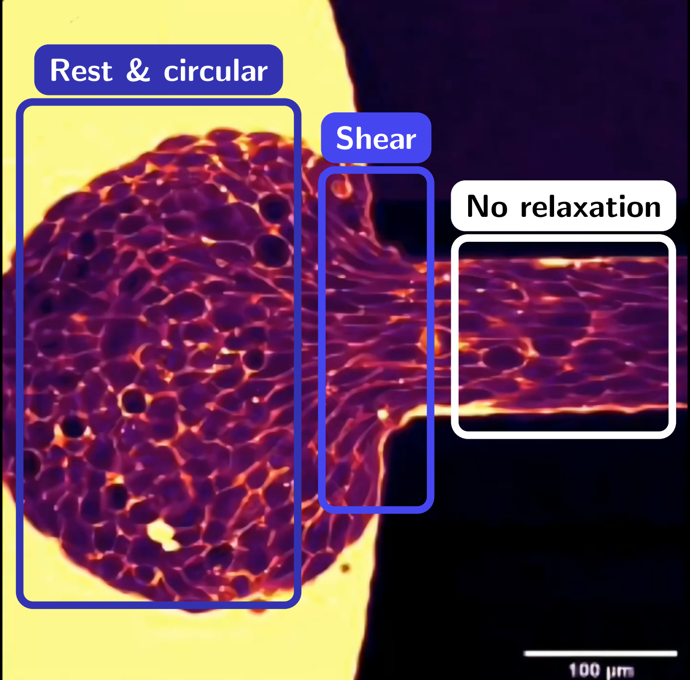



100 μm


Rest & circular
100 µm



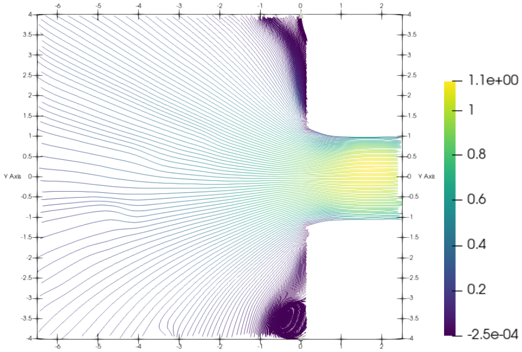
Rest & circular
Shear
100 µm



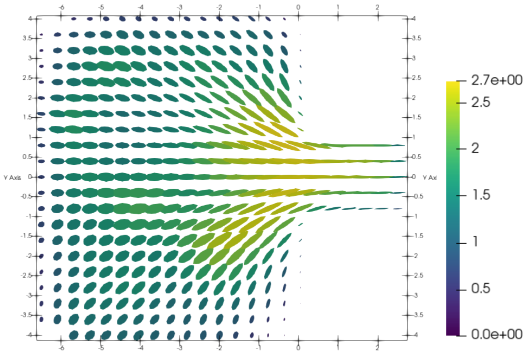
Rest & circular
Shear
No relaxation
100 μm

Experimental data fields

Processed from *[Tlili et al. 2022]*



Velocity streamlines



Conformation field

Identify: Cell shape $\longleftrightarrow$ Ellipse

Navier-Stokes is not enough !

Modeling needs

Unknowns

Pressure	p	Non-Newtonian
Velocity	$\boldsymbol{u}$	
Conformation	$\boldsymbol{c}$	

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Parameters

Viscosity ratio	$\alpha \in [0,1]$
Weissenberg	$\mathrm{We} \in \mathbb{R}_+$
Reynolds	$\mathrm{Re} \in \mathbb{R}_+$

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Assumptions

1. Isotropic rest state
2. $\mathrm{Re} \ll 1$ (small velocity)
3. Depth invariance $\rightarrow$ 2D

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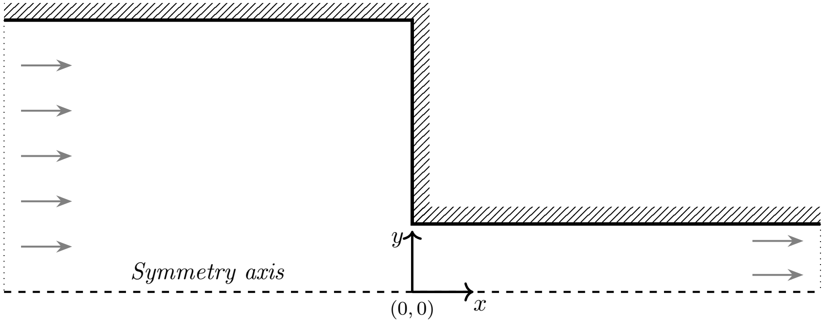
Features

1. Stationary solutions
2. Slip boundaries
3. Abrupt geometry

Geometry
Slip boundary conditions



Oldroyd-B model

Adapted from *[Oldroyd 1950]*

Find $\boldsymbol{c}, \boldsymbol{u}$ and p defined in Ω such that

$$\begin{cases} \overset{\nabla}{\boldsymbol{c}} &= -\dfrac{1}{\mathrm{We}}\left(\boldsymbol{c} - \mathbb{I}\right) \\ -\mathrm{div}\left(\boldsymbol{\sigma}\right) &= 0 \\ \mathrm{div}\left(\boldsymbol{u}\right) &= 0 \\ \boldsymbol{\sigma} &= \dfrac{\alpha}{\mathrm{We}}\left(\boldsymbol{c} - \mathbb{I}\right) + 2(1-\alpha)D(\boldsymbol{u}) - p \cdot \mathbb{I} \end{cases}$$

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\end{cases}
$$

Assets

- Rest state: $\boldsymbol{c} = \mathbb{I}$
- Linear, large deformations

Limits

- No global existence of solutions
- Unbounded extension with $\mathrm{Tr}(\boldsymbol{c})$

FENE-P model

Adapted from *[Bird et al. 1980]*

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$$\left\{\begin{array}{rcl}
\overset{\triangledown}{\boldsymbol{c}} & = & -\dfrac{1}{\mathrm{We}}\left(\dfrac{\boldsymbol{c}}{1-\beta\,\mathrm{Tr}\,\boldsymbol{c}} - \dfrac{\mathbb{I}}{1-\beta\,d}\right) \\
-\mathrm{div}\,(\boldsymbol{\sigma}) & = & 0 \\
\mathrm{div}\,(\boldsymbol{u}) & = & 0 \\
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\end{array}\right.$$

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Extension bound

$$\mathrm{Tr}(\boldsymbol{c}) < \frac{1}{\beta}$$

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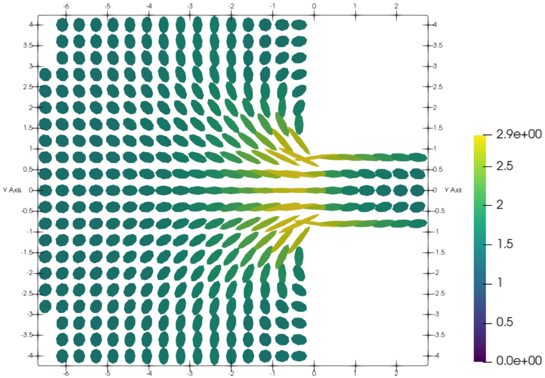
Assets

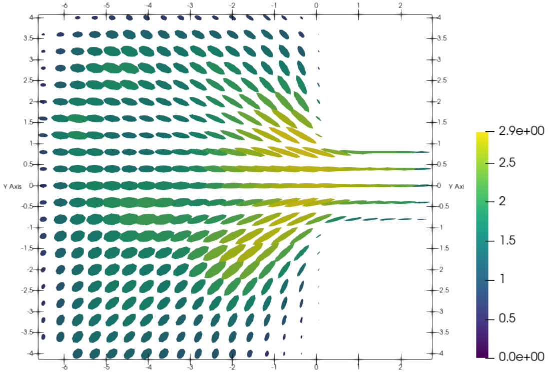
- Bounded extension
- Large deformations
- Global (weak) existence *[Masmoudi 2011]*
- Contains Oldroyd-B ($\beta = 0$)

FENE-P vs. Experiments

Conformation maps



FENE-P



[Tlili et al. 2022]

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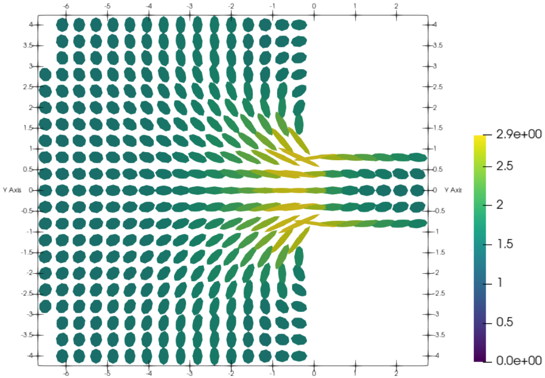
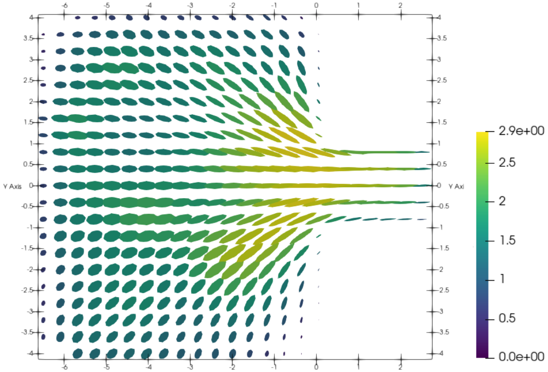


FENE-P



[Tlili et al. 2022]

Upstream & Entrance ✓

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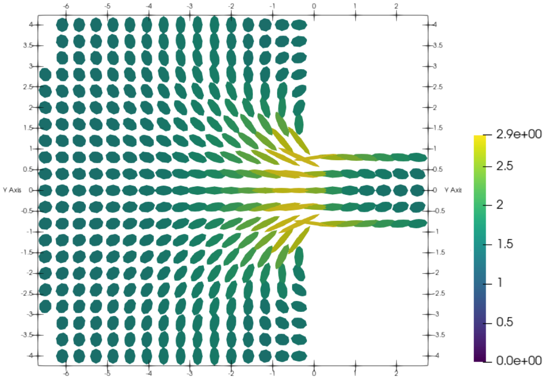
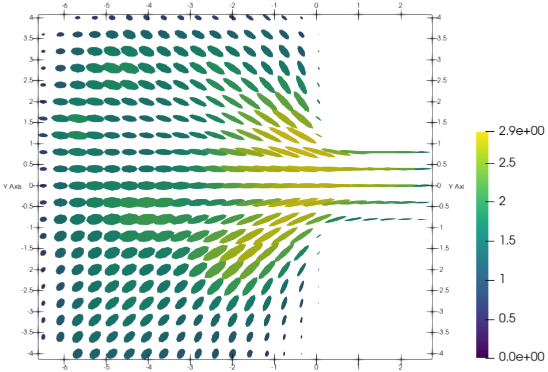


FENE-P

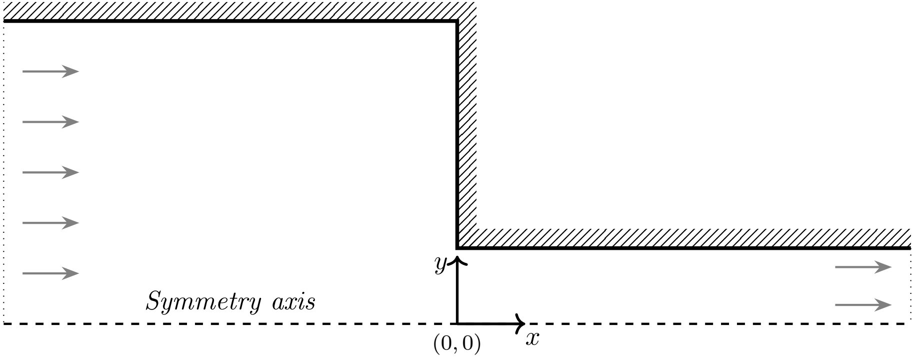


[Tlili et al. 2022]

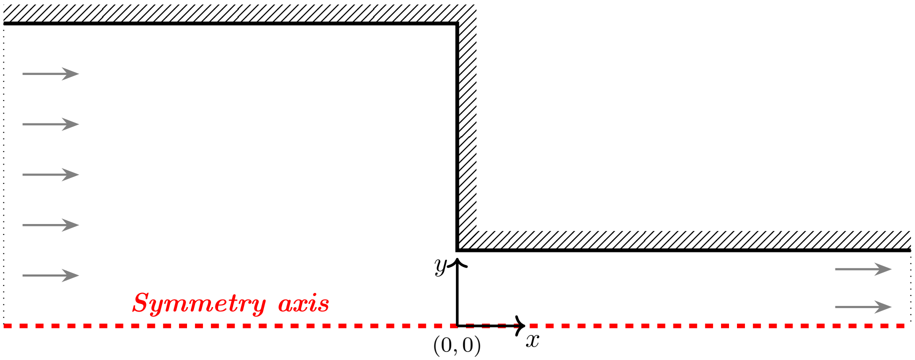
Upstream & Entrance ✓

Downstream relaxation not matching !

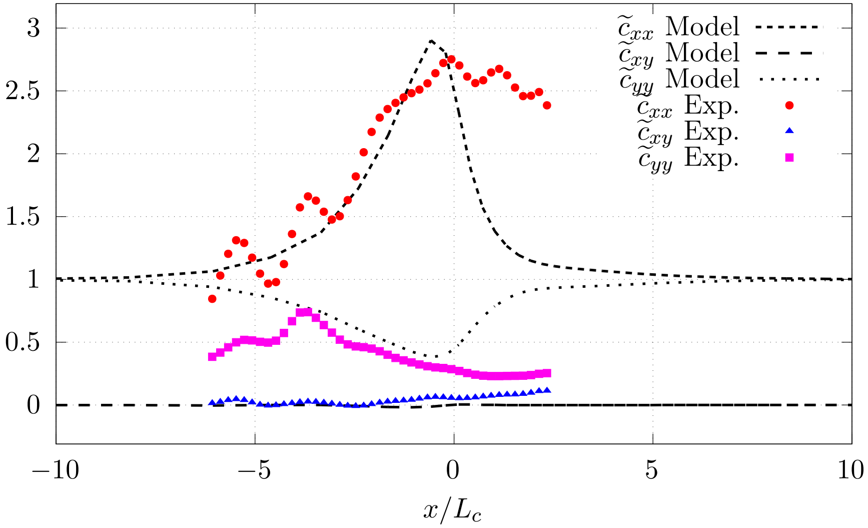
Symmetry axis



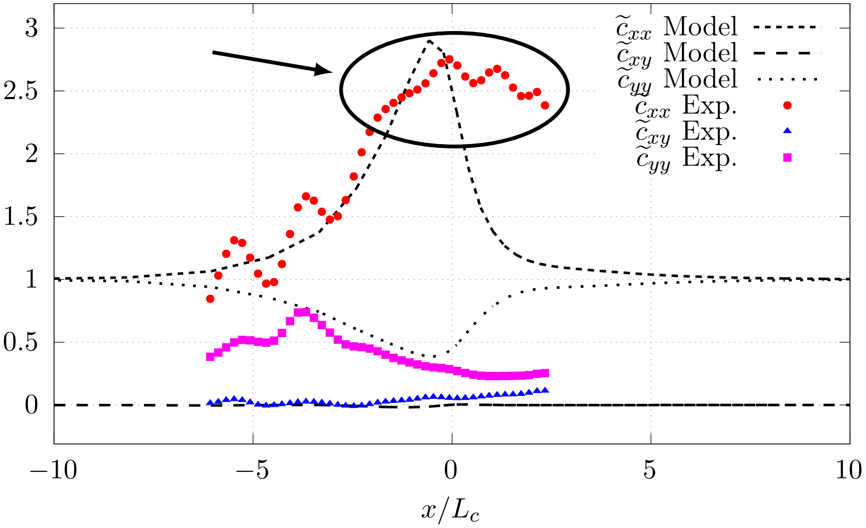
Symmetry axis



Conformation cut

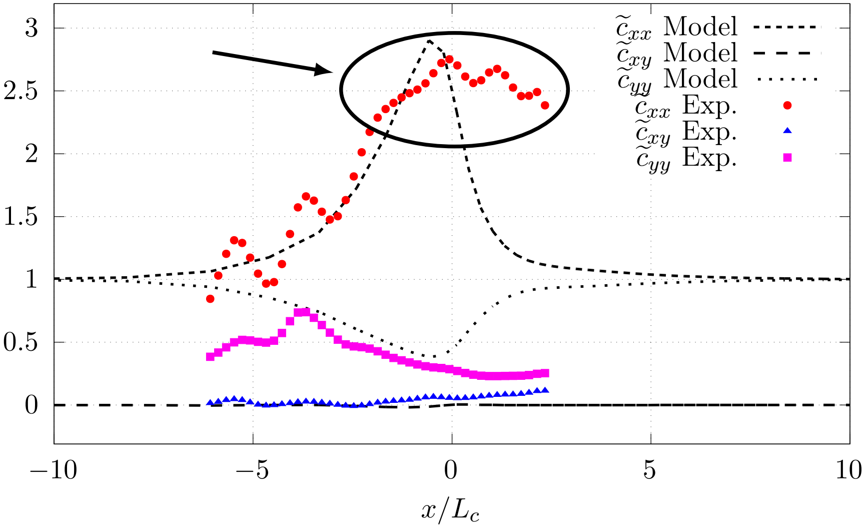


1. Good upstream



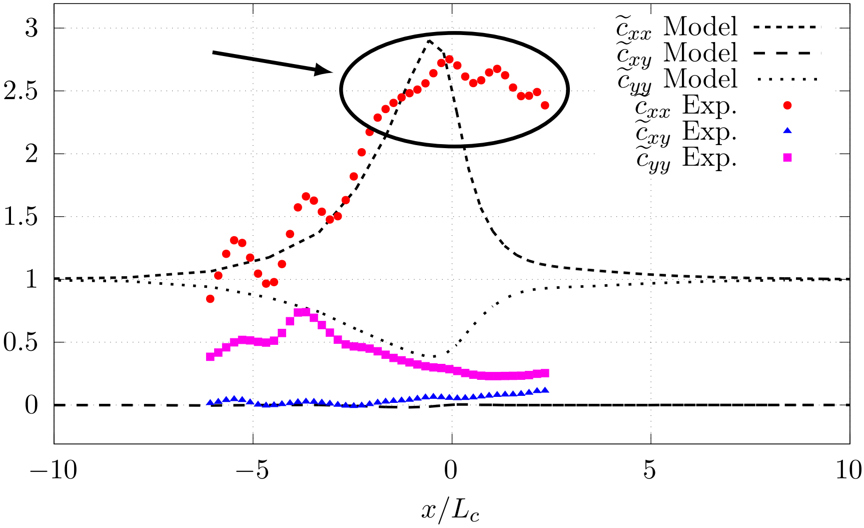
1. Good upstream
2. Downstream plug

Conformation cut



1. Good upstream
2. Downstream plug
3. Unfeasible larger *We* number

Conformation cut



1. Good upstream
2. Downstream plug
3. Unfeasible larger *We* number

Yield stress needed !

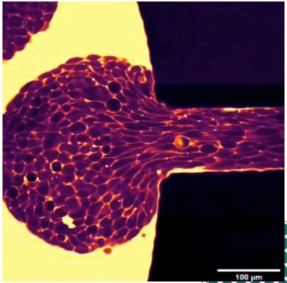
Conclusion

Advances

- Embryonic tissues: viscoelastic behavior
- Oldroyd-B: unbounded extension
- FENE-P: upstream and entrance ✓
- **Agreement for large deformation viscoelasticity**

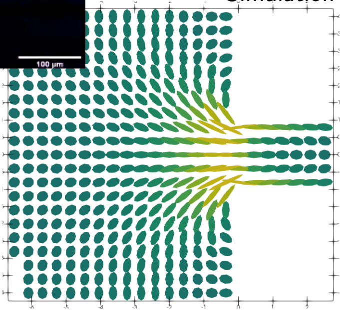
Perspectives

- **New model** with elasticity & yield stress
- Downstream **channel study**
- **Mutants** assessment



Experiment [Tlili et al. 2022]

Simulation



Thank you for listening !

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